

Analysis of Students' Prior Ability in Mathematical Logical Thinking Ability

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Abstract: The aim of this article is to analyze mathematics education students' prior ability in mathematical logical thinking ability. Research is conducted in third semester in one of universities in Asahan, the material tested to the students is composition of function with mathematical logical thinking indicators. It was found that out of 28 students who take the test, more than half of the students' answer less than four problems, and also from four problems they solve, there are students whose answers are still poor and inaccurate, it is shown that student prior ability in logical thinking ability is still lower.

Key words: *logical thinking ability, prior ability*

INTRODUCTION

The goal of learning mathematics is to prepare students to be able to deal with changing circumstances in life and in an ever evolving world, through practice acting on the basis of logical, rational, critical, accurate, honest, efficient and effective thinking [1]. The other goal is to mastering mathematics presents specific requirements to the level of the students' logical reasoning [2]. Students' logical thinking ability needs to be improved. Students generally think in algorithm, but that is not logical thinking as there is difference between logical thinking and just thinking [3]. It is a teacher task to find the right way to teaching so the result will give positive effect to students' logical thinking abilities. To be able to teach properly, teacher must have good logical thinking ability as well.

Considering that the students of mathematics education are mathematics teacher candidates who will act as educators as well as math teachers in the future, they must be prepared with competencies put forward by the government, and need to be equipped with sufficient experience and better knowledge, one of the competency that should be given to them is mathematical logical thinking ability.

Thinking activities begin when there are doubts and questions to answer or deal with issues or problems that require problem solving [4], thinking can be said a process to find a true truth or knowledge by involving knowledge or experience possessed [5].

The ability to think logically is the ability to find a truth based on certain rules, patterns or logic [6]. By having

logical thinking ability students can have the ability to acquire, manage, and utilize information. In addition, by logical thinking students are trained to think scientifically in order to survive in an ever-changing, uncertain, and increasingly competitive state. The ability to think logically gives students the ability to understand what they read or learn. Logical thinking also encourages students to think, propose hypotheses, develop alternative hypotheses, and test their hypotheses based on known facts, to draw conclusions [7]. This is evident from the questions about logical thinking where in each answer there is a logical reason that accompanies it [8]. It means learning logically enables them to understand the situation and find a logical solution that leads them towards logical thinking. This capability needs to be developed in the learning of mathematics, because it can help students to improve the ability of understanding mathematics [9-10].

Based on the explanation above, it is important to know the condition of students' mathematical logical thinking ability particularly their prior ability, so educators can find the right way to help students mathematical logical thinking ability to increase positively.

RESEARCH METHODS

The method of this study is qualitative by using descriptive type to analyse students' prior ability of mathematical logical thinking skill. In this study, the subject is twenty eight students in third semester at one of universities in Asahan. The instruments in this

research are a test which includes mathematical logical thinking ability indicators and interview. The indicators of mathematical logical thinking ability used in this research are;

1. drawing conclusion, estimates, and interpretations based on appropriate proportions which means students can draw conclusions by solving a problem that is not yet known one of its components based on available component
2. drawing conclusions or making predictions based on probabilities means students can draw conclusions and make estimates of a problem based on some information related to the problem
3. drawing conclusions, estimates, and predictions based on the correlation between two variables which means students can draw conclusions, estimates, and predictions of a problem based on the correlation between the two variables contained in the problem.
4. proving or constructing evidence means students can prove a problem by providing the appropriate facts or constructing evidence from the information provided by the question
5. compiling analysis and synthesis of some cases means students can check the correctness of the relationship from two or more cases of a problem based on information provided by the question
6. drawing conclusions or estimates based on the similarity of two processes means students can draw conclusions by examining the similarities or relationships of the two processes (analogy)
7. defining a combination of several variables which means students can define a value or process by using a combination of some information or answer from process done before. [9]

RESULT AND DISCUSSION

The research conducted in third semester in one of universities in Asahan, the material that tested to the students is composition function with mathematical logical thinking indicators used in the prior ability test: (1) drawing conclusion, estimates, and interpretations based on appropriate proportions; (2) draw conclusions or make predictions based on probabilities; (3) drawing conclusions, estimates, and predictions based on the correlation between two variables; (4) proving or constructing evidence; (5) compiling analysis and synthesis of some cases; (6) drawing conclusions or estimates based on the similarity of two processes (analogy); and (7) defining a combination of several variables. It is found that from 28 students who take the test, more than half of the students' answer less than four problems, and also from four problems that they solve, there are many students whose answers are still poor and inaccurate and students' prior ability of logical thinking ability is still low.

From seven problems given, more than half of the students did not answer more than four problems. Even some of them only answer one or two questions. From the answer sheet can be seen some student understand the problem and many is not.

Problem that all students answer is problem no 7 with the indicator is drawing conclusions, estimates, and predictions based on the correlation between two variables. From 28 students who answer this problem only 10 students get the right answer. From 18 students whose answers are poor and inaccurate, some of them understand the problem and some do not. Students who do not understand without looking for the inverse value in question directly do the calculation to find "a" value by entering the number 3 as a result to a given function, and the other of them insert the number 3 which is the inverse result not the value of "a" into a given function. Students whose understand the problem, do find the invers function but put number 3 as "a" value not as the invers function result, as shown in picture below:

Handwritten student work for problem no. 7:

$$f(x) = \frac{3x+2}{x-5}$$

$$y = \frac{3x+2}{x-5}$$

$$y(x-5) = 3x+2$$

$$xy - 5y = 3x+2$$

$$2y - 3x = 5y+2$$

$$x(y-3) = 5y+2$$

$$x = \frac{5y+2}{y-3}$$

$$f'(a) = \frac{5a+2}{a-3}$$

$$f'(3) = \frac{5(3)+2}{3-2}$$

$$f'(3) = \frac{15+2}{1}$$

$$f'(3) = 17$$

Figure 1 Student answer for problem no.7

Handwritten student work for problem no. 7:

$$f'(a) = 3$$

$$f'(a) = 3$$

$$= 3 \cdot 3 + 2$$

$$\frac{9+2}{3-2}$$

$$= \frac{11}{1}$$

$$\frac{1}{a}(a) = \frac{1}{-2} \quad \text{Jadi } a = \frac{11}{-2}$$

Figure 2 Student answer for problem no.7

After conducting interview to the students who the answer is chosen above, one student who do find the inverse function but put 3 as a value of "a" said that he

forgot that the question asking the “a” value, because usually the problem he has ever done always asking the value of the function if it is known that the value of x equals to 3. It is clear that students are accustomed to working on the problem regardless of the process of thinking that is formed.

The other problem that many students answer is problem number 3, the indicator is drawing conclusion, estimates, and interpretations based on appropriate proportions. From 23 students who answer this problem, 9 students get the right answer, 2 students only answer one part and 12 students answer this problem incorrectly. Student that answer incompletely shown that actually they understand the problem, but maybe they think the problem only one question or they forgot to the other question. For students which answer incorrectly, the mistake of their answer shown in “a” part they do the wrong calculation, as the “b” and “c” question can be calculated after finding the “a” question so the answer of “b” and “c” question also wrong. Students find the g (x) value of the information from gof(x), by putting the f(x) function to the gof(x) function that is why student miscalculate so their answer is wrong. This indicates that the students’ ability to estimate a function’s value based on a composition of function whose value and one of the other functions are known is still low, students are still lack in terms of performing calculations to be adjusted to the information provided. This shown that students logical thinking ability in interpretations indicator based on appropriate proportions is still poor, as shown in following picture:

3. $f(u) = u + 2$
 $(g \circ f)(u) = u^2 + 3u - 5$
 a. $g(u) = (u+2)^2 + 3(u+2) - 5$
 $= u^2 + 4u + 4 + 3u + 6 - 5$
 $= u^2 + 7u + 5$
 b. $u = a$
 $g(a) = a^2 + 7a + 5$
 c. $g(u-1) = u^2 + 7u + 5$
 $= (u-1)^2 + 7(u-1) + 5$
 $= u^2 - 2u + 1 + 7u - 7 + 5$
 $= u^2 + 5u - 1$

Figure 3 Student answer for problem no.3

3. Dik: $f(x) = x + 2 \rightarrow x + 2 = a$
 $x = a - 2$
 $(g \circ f)(x) = x^2 + 3x - 5$
 dit: a) $g(x)$
 b) $g(a)$
 c) $g(x-1)$
 Penyelesaian
 a) $(g \circ f)(x) = x^2 + 3x - 5$
 $g(f(x)) = x^2 + 3x - 5$
 $g(x+2) = x^2 + 3x - 5$
 $g(a) = (a-2)^2 + 3(a-2) - 5$
 $g(a) = a^2 - 4a + 4 + 3a - 6 - 5$
 $g(a) = a^2 - 7a + 5$
 b) $g(x) = x^2 + 3x + 5$
 c) $g(x-1) = x^2 + 3x - 7$
 $= (x-1)^2 + 3(x-1) - 7$
 $= x^2 - 2x + 1 + 3x - 3 - 7$
 $= x^2 + 5x - 9$

Figure 4 Student answer for problem no.3

Dik: $f(u) = u + 2$
 $(g \circ f)(u) = u^2 + 3u - 5$
 Dit: a. $g(u)$
 b. $u = a$ nilai $g(u)$
 c. $g(u-1)$
 Jawab: Misal: $u + 2 = a \Rightarrow u = a - 2$
 a) $(g \circ f)(u) = u^2 + 3u - 5$
 $g(f(u)) = u^2 + 3u - 5$
 $g(u+2) = u^2 + 3u - 5$
 $g(a) = (a-2)^2 + 3(a-2) - 5$
 $= a^2 - 4a + 4 + 3a - 6 - 5$
 $= a^2 - a - 7$
 $g(u) = u^2 - a - 7$
 b) $u = a \Rightarrow g(a) = a^2 - a - 7$
 c) $g(u-1) = (u-1)^2 + 3(u-1) - 7$
 $= u^2 - 2u + 1 + 3u - 3 - 7$
 $= u^2 + 3u - 9$

Figure 5 Student answer for problem no.3

From the interview with one student who answer correctly, it was found that students have often done problems like this, it shows that students have not realized that when solving problems like this student are trained to think logically, students just feel the need to memorize how to answer the problem without understanding the problem deeper, or what benefits can students receive after solve the problem. The same thing also applies to students who answered incorrectly, the students said they had often working on a problem like this while in high school but they forgot how to do it. It shows that the learning they got before is not engrave in their memory, because they used to answer question not solve the problem.

The next problem that students do is problem number 5 with indicator drawing conclusions or estimates based on the similarity of two processes (analogy). Only 9 students answer this problem. From 9 students only 3 students answer completely. Students who answer completely have not linked the reasons they provided in accordance with the information known, the students simply said that because the result of fog (x) inverse exists in the form of the root then the result is ambiguous, because the result value will be in negative or positive. It shown that actually students started to

think logically, but students cannot link the similarities process that has been described in the problem that domain can only be mapped once in order to become a function, while the form of $f \circ g(x)$ inverse found in the root form, there will be two mappings so it cannot be said as a function. As shown in the following picture:

$f(x) = 2x + 1$
 $g(x) = x^2$
 Dit: $(f \circ g)^{-1}$
 Jawab
 $(f \circ g) = 2g(x) + 1$
 $= 2(x^2) + 1$
 $= -2x^2 - 1$
 $(f \circ g)^{-1}$
 $y = -2x^2 - 1$
 $2x^2 = -y - 1$
 $x^2 = \frac{-y - 1}{2}$
 $x = \sqrt{\frac{-y - 1}{2}}$

Figure 6 Student answer for problem no.5

$f(x) = 2x - 1$
 $g(x) = x^2$
 apakah $f \circ g$ merupakan fungsi? \rightarrow
 $h(x) = (f \circ g)(x)$
 $h(x) = f(g(x))$
 $= f(x^2)$
 $= 2(x^2) - 1$
 $= 2x^2 - 1$
 $(f \circ g)^{-1}$
 $y = 2x^2 - 1$
 $-2x^2 = -y - 1$
 $2x^2 = y + 1$
 $x^2 = \frac{y + 1}{2}$
 $x = \sqrt{\frac{y + 1}{2}}$
 Alasan
 Karena fungsi itu bernilai dua, yang dimana jika bernilai dua nilai, bisa mendapat hasil yg sama, bisa +, -

Figure 7 Student answer for problem no.5

From interviews conducted to students who answered with reason said that they understand the result of $f \circ g(x)$ inverse in the form of the root mean there are two possibilities answer, but codomain of function only mapped once so it is not a function. Students who do not give a reason also say that because the root result in a root shape so it is not a function. Students understand the thinking process but still not relate it to the information and process given in the problem, also they say that they are not accustomed to answer the problem by providing an appropriate or logical reason. This shows that students have not been trained to have logical thinking when working on a problem.

For problem number 4, the indicator is compiling analysis and synthesis of some cases. This indicator state that students can check the correctness of the relationship from two or more cases of a problem based on information provided by the question. Only 8

students that answer this question, 4 students answer correctly and the other four is not. the errors contained in the student's answer are they are wrong in determining the inverse value of each function in the problem, and also the student is wrong in calculating the result of the composition function of the 3 functions given in question, as seen in the following picture:

$(g \circ f)(h(x))^{-1} = (h^{-1} \circ f(g(x)))^{-1}$
 $f(x) = 4x + 1$
 $g(x) = 3(4x + 1) + 2$
 $= 12x + 3 + 2$
 $= 12x + 5$
 $g(x)^{-1} = \frac{x - 5}{12}$
 tidak sama
 $(g \circ f)(x) = x + 1$
 $f(x) = \frac{x + 1}{4}$
 $(g \circ f \circ h)(x) = g(f(h(x)))$
 $= 3(4x + 2) + 2$
 $= 12x + 6 + 2$
 $= 12x + 8$
 $(g \circ f \circ h)^{-1}(x) = y = 12x + 8$
 $-12x = -y + 8$
 $12x = y - 8$
 $x = \frac{y - 8}{12}$

Figure 8 Student answer for problem no.4

$(g \circ f \circ h)(x) =$
 $f(h(x)) = x + 1$
 $f(4x) = 4x + 1$
 $(g \circ f \circ h)(x) = g(f(h(x)))$
 $= 3(4x + 2) + 2$
 $= 12x + 6 + 2$
 $= 12x + 8$
 $(g \circ f \circ h)^{-1}(x) = y = 12x + 8$
 $-12x = -y + 8$
 $12x = y - 8$
 $x = \frac{y - 8}{12}$

Figure 9 Student answer for problem no.4

$f(x) = y - x + 1$
 $-x = -y + 1$
 $x = y - 1$
 $x - 1 = y - 2$
 $x = \frac{y - 2}{3}$
 $h(x) = y - 4x$
 $4x = y$
 $x = \frac{y}{4}$
 $(f \circ g)^{-1}(x) = f^{-1}(g(x))$
 $f^{-1}\left(\frac{y - 2}{3}\right)$
 $\frac{y - 2}{3} - 1$

Figure 10 Student answer for problem no.4

From interviews to students who answer correctly, they said that they had done such a problem. For the student who does the answer is not quite right, say that he is in a hurry because he wants to do the other problems. Another student says that she forgot the right way so she just answers as much as she can about it. While other students who do not answer said that because there are

3 functions that will be calculated to the value of the composition so it will be the long process, so they choose to work on other problems first, if the time is still available then they will try to do the problem. Another student said that he forgot about the function of the composition with three functions so he not answers it at all. This indicates that the lesson learned by the students before is not so remembered by them, this can be happened because students are accustomed to answer problems that only aim for value, but do not build students' thinking skill.

Problem number 1 with indicator proving or constructing evidence, only 6 students do answer the problem, with 3 students answer correctly and completely while other 3 only answer one part. Students did not answer the problem completely maybe because they did not remember or understand the way to solve the problem given. The students answer shown in picture below:

$$1. a. g(x) = \frac{-dx + b}{cx - a}$$

$$f(x) = \frac{ax + b}{cx + d}$$

$$y = \frac{ax + b}{cx + d}$$

$$cxy + dy = ax + b$$

$$cxy - ax = b - dy$$

$$x(cy - a) = b - dy$$

$$x = \frac{b - dy}{cy - a}$$

$$f^{-1}(y) = \frac{b - dy}{cy - a}$$

Figure 11 Student answer for problem no.1

$$a) y = \frac{ax + b}{cx + d}$$

$$ycx + yd = ax + b$$

$$ycx - ax = b - yd$$

$$x(cy - a) = b - dy$$

$$x = \frac{b - dy}{cy - a}$$

$$f^{-1}(x) = \frac{b - dx}{cy - a}$$

$$b) f(g(x)) = I(x)$$

$$\frac{a(g(x) + b)}{c(g(x) + d)} = x$$

$$\frac{a(\frac{b - dx}{cx + a} + b)}{c(\frac{b - dx}{cx + a} + d)} = x$$

$$\frac{ab - adx + bcx + ab}{cb - cdx + dcx - ad} = x$$

$$\frac{-adx + bcx}{cb - ad} = x$$

$$x(bc - ad) = x(bc - ad)$$

(terbukti)

Figure 12 Student answer for problem no.1

From the results of interviews with students who worked on the problem said that they had done such a problem before. While the reason students only do part "a" because after try to solving the problem on another paper

before they did not find the results, and said that they miscalculated so they did not write the answer and chose to do another problem. Other students say that it seems like long to solve it so she chooses to do another problem. Students who did not answer questions said they had never done such a thing so chose not to do it at all. Another reason the student said she had seen such a problem but did not remember how to do it so chose not to work on the problem. This shows that most students are still not good in thinking skills, as has been said before, this may happen because students are accustomed to working on problems that seek a certain value rather than working on a question that asks students to do reasoning, proving or interpreting. Though such questions can help them build their thinking ability.

Other problem that only few students answered is problem number 6, the indicator is define a combination of several variables. This problem actually seen as easy one, but only 5 students answer this problem and only one got the closes answer to the right answer. students who are wrong in answering only focus on a given function but are not focused on the question so they cannot use their knowledge of the mapping in the function. Another mistake that arises is the student forgot to enter one other pair so that the result is formed into a function. As seen in the picture:

$$b) A = \{3, 2, 1, -1\}$$

$$B = \{2, 3, 5\}$$

$$f: A \rightarrow B$$

$$f = \{(3, 2), (2, 3), (1, 5), (-1, 5)\}$$

$$f: A \rightarrow B$$

$$= \{(3, 2), (2, 3), (1, 5), (-1, 5)\} \Rightarrow \text{merupakan sebuah fungsi}$$

$$\text{Maka: } f^{-1}(x) = f: B \rightarrow A$$

$$= \{(2, 3), (3, 2), (5, 1)\}$$

Karena semua B dapat dipetakan, maka $f^{-1}(x)$ tersebut adalah sebuah fungsi juga.

Figure 13 Student answer for problem no.6

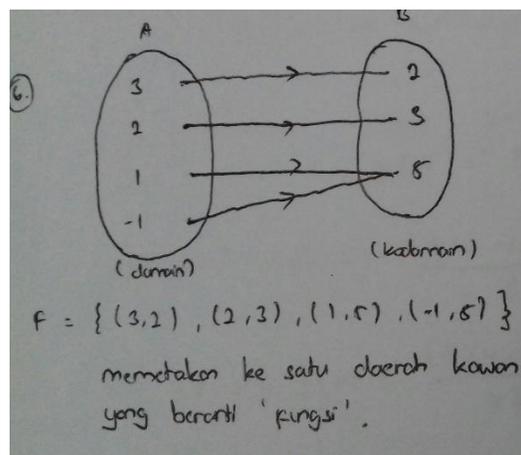


Figure 14 Student answer for problem no.6

From the interviews to the students who answered almost rightly, she understood the meaning of the mapping but forgot to include the one pair so that the result became a function, the student said that he should not function because the domain is mapped twice to the codomain. This indicates that the student has been able to establish a combination of several variables in this case is to define two sets of mapped points by searching for inverse functions. The results of interviews with students who do not work on this problem say that many students forget about the mapping so choose to do another problem. Other students say because they are focused on doing other things that do more calculations so that when time is up the students do not have time to work on this problem. Again this shown that the students are used to working on the problem without understanding and thinking why this problem needs to be done and what the benefits of working on the problem so they easily forgot the way to solve the problem.

One problem that all students did not answer is problem no 2 with indicator drawing conclusions or making predictions based on probabilities. It is expected that students can draw conclusions and make estimates of a problem based on some information related to the problem. This happens maybe because students avoid solving the problems which demand reasoning or describing relationship between the results that found.

From the interview result it was found that students do not work on this problem because they rarely do such a thing, they choose another problem that is easier to do. The other students said that this question asks a graph and it will take time to draw it so they prefer to work on another problem. The other students also said this problem asks them to give reason, while they do not used to it, they said that they confused what to write and explain to this kind of question, so they choose not to work on the problem at all. From all the explanation above it is clear that students' mathematical logical thinking ability is still low.

CONCLUSION

More than half students' logical thinking ability is still low. Half of the students still find hard to solve the problem which needs prove, statements, reasons, or

explanations that support their answer. Hence, the teacher should apply the innovative instructional method to enhance the students' mathematical logical thinking ability and create problems that support students to improve their thinking skills, especially their logical thinking abilities by giving them non-routine test to develop their mathematical concepts as well as logical thinking skill.

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